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JAP-003-0491002

Seat No. ______

B. Sc. / M. Sc. (Applied Physics) (Sem. I) (CBCS) Examination

November - 2019

Mathematics

(Fundamentals of Mathematics) (New Course)

Faculty Code : 003

Subject Code : 0491002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures on in right side indicate marks.

1 Write answers of any **seven** questions : **14**

(1) If $A=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 1 & 1\\ -1 & -1 & -1\end{pmatrix}$ then find AB.

(2) Find the trace of $\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$.

(3) If $A=\begin{pmatrix}4 & 2\\ 3 & 1\end{pmatrix}$ then find adj A.

(4) Find the polar form of $1+i$.

(5) If $A=1+2i$ and $B=2-i$ then find AB.

(6) Evaluate $\lim\limits_{x\to 0}\dfrac{\tan mx}{\sin nx}$.

(7) If $t=u^5$ then find $\dfrac{du}{dt}$.

(8) Evaluate $\int x\log x\,dx$.

(9) Find the angle between the vectors (1, –1) and (–1, 1).

(10) If $\overrightarrow{OA}=(1,2)$ and $\overrightarrow{OB}=(2,1)$ then find the length of $\overrightarrow{AB}$.

2 (a) Write answers of any **two** questions : **4**

(1) If A is a square matrix then prove that $A - A'$ is a skew-symmetric matrix.

(2) If A and B are orthogonal matrices, then prove that $AB + BA$ is an orthogonal matrix.

(3) If A is an idempotent matrix, then prove that $I - A$ is an idempotent matrix.

(4) Express $\begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$ as sum of a symmetric and a skew-symmetric matrix.

Hint : $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

(b) Write answers of any two questions : **10**

(1) Using row operations find the inverse of $\begin{pmatrix} 2 & 3 & 5 \\ 1 & 2 & 7 \\ 3 & 4 & 4 \end{pmatrix}$.

(2) Find the rank of $\begin{pmatrix} 7 & 5 & 2 \\ 4 & 7 & 9 \\ 3 & 6 & 8 \end{pmatrix}$.

(3) If w is an imaginary cube root of unity and

$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$ then prove that $|A| = \sqrt{3}i$.

(4) Solve : $2A + B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $A + 2B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ where $A, B \in M_2$.

3 (a) Write answers of any **two** questions : **4**

(1) If w is an imaginary cube root of unity then prove that $1+w+w^2=0$.

(2) Simplify : $\frac{1-i}{2+i}$.

(3) Find the real part of exp $\left(2+\frac{3\pi}{4}i\right)$.

(4) Prove that $\left\{\cos(\alpha-\theta)-e^{-i\alpha}\cos\theta\right\}^n=i^n\sin^n\alpha\, e^{-in\theta}$.

(b) Write answers of any **two** questions : **10**

(1) Evaluate $\sum_{n=1}^{100} i^n$.

(2) If $\cos\alpha+\cos\beta=\sin\alpha+\sin\beta=0$, then prove that $\cos 3\alpha+\cos 3\beta=\sin 3\alpha+\sin 3\beta=0$.

(3) Simplify $\left(1+\sqrt{3}i\right)^{-5}+\left(1-\sqrt{3}i\right)^{-5}$.

(4) If $x_n=cis\frac{\pi}{2^n}$ then prove that $\prod_{n=1}^{\infty}x_n=-1$.

4 (a) Write answers of any **two** questions : **4**

(1) Evaluate $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$.

(2) Prove that $f(x)=|x|$ is not differentiable at $x=0$.

(3) Prove that the function $y=\cos^2 x$. Satisfies the relationship y''' + 4y' = 0

(4) Evaluate $\int_0^{\pi} x\cos^2 x\,dx$.

Hint : $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$.

(b) Write answers of any two questions : **10**

(1) Evaluate $\lim_{x \to 0} \left(\frac{1}{e^x - 1} - \frac{1}{x} \right)$.

(2) Let $f(x) = x + 1$ when $x \le 1$

$= 3 - x^2$ when $x > 1$

then prove that f is continuous at $x = 1$.

(3) Prove that $\left(\cos^n x \cos nx\right)' = -n \cos^{n-1} x \sin(n+1)x$.

(4) If $f(n) = \int_0^{\pi/4} \tan^n x \, dx$ then prove that

$f(n) + f(n-2) = \frac{1}{n-1}$ for $n \ne 1$. Deduce $f(3)$.

5 (a) Write answers of any **two** questions : **4**

(1) If $a = (1, 2)$, $b = (2, 3)$, $c = (3, 4)$ then find

$\frac{1}{2}(a - 2b + 3c)$.

(2) Find a unit vector perpendicular to the plane containing the vectors (1, 2, 3) and (3, 2, 1).

(3) If $a = (1, -1, 1)$, $b = (-1, 1, 1)$ and $c = (1, 1, -1)$ then evaluate $[abc]$.

(4) If $a = (1, 2, 4)$, $b = (3, 5, 1)$ then evaluate $(a + b)\cdot(a - b)$.

(b) Write answers of any **two** questions : **10**

(1) Simplify $[b + c, c + a, a + b]$.

(2) If a, b, c are mutually perpendicular vectors of equal magnitude then show that vector $a + b + c$ is equally inclined to a, b, c.

(3) If $a = (1, 2, 1)$, $b = (2, 1, -1)$, $c = (-1, 1, 1)$ then evaluate $(a \times b)\cdot(b \times c)$.

(4) If $a = (1, 2, 3)$, $b = (2, 3, 1)$, $c = (3, 1, 2)$ then evaluate $a \times (b \times c)$.